

A Viral Marketing-Based Model For Opinion Dynamics in Online Social Networks

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Facebook data: How it was used by Cambridge Analytica

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(BBC)

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Who's a Bot? Who's Not?

It sometimes seems that automated bots are taking over social media and driving human discourse. But some (real) researchers aren't so sure.

(The New York Times)



Emmanuel Macron
Republic on the Move



Marine Le Pen
National Rally

Polarizing vs Marketing Content



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Polarizing content: like it or hate it

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Polarizing content: like it or hate it



Marketing content: like it or ignore it

What Do We Need to Understand?

- How does the content spread through the network?
- When exposed to new content, how do people form their opinions?
- How does spreading content affect the polarization of the network?

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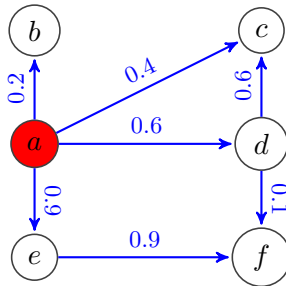
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Existing Models

■ Independent Cascade Model (Kempe, Kleinberg, Tardos; 2003)

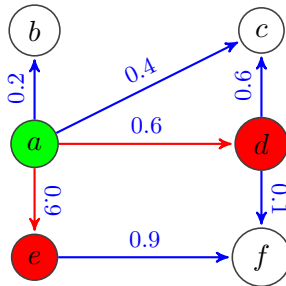
- Models how viral content spreads in social networks
- Each node has one shot to influence its neighbors
- Once activated, nodes stay activated



a initially gets activated at time step 0.

■ Independent Cascade Model (Kempe, Kleinberg, Tardos; 2003)

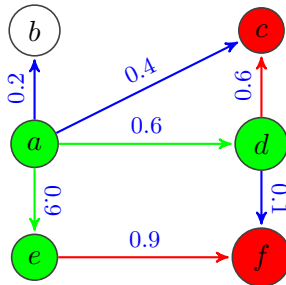
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At time step 1, a activates d and e , but fails to activate b and c .

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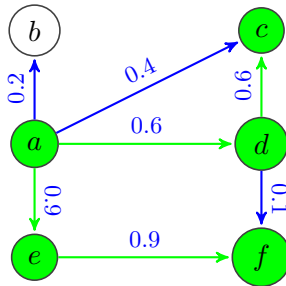
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At time step 2, d activates c , e activates f

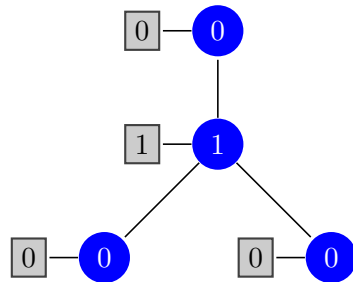
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At time step 3, c and f don't have neighbors, the spreading stops here.

- Friedkin–Johnsen model (FJ model) (Friedkin, Johnsen; 1990)
 - Each node i has **innate** and **expressed** opinions
 - **innate** opinion $s_i \in [0, 1]$: fixed, kept private
 - **expressed** opinion $z_i^t \in [0, 1]$: depends on time t , public

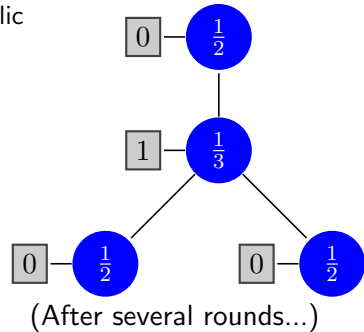


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Update rule

$$z_i^{t+1} = \frac{s_i + \sum_{j \in N(i)} w_{i,j} z_j^t}{1 + \sum_{j \in N(i)} w_{i,j}}$$

with edge weights $w_{i,j}$ and $N(i) =$ neighbors of i

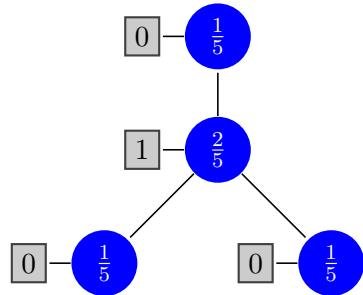


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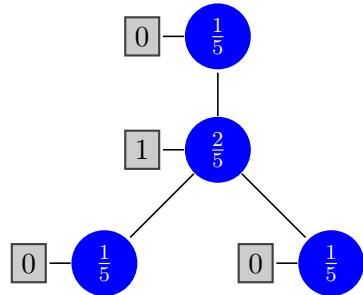
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After convergence $\mathbf{z} = (\mathbf{I} + \mathbf{L})^{-1} \mathbf{s}$,
where \mathbf{L} is the graph Laplacian.



- Existing models focus on **either** information spreading **or** opinion formation
- To understand how viral content influences user opinions, we need **both**

⇒ Can we get a model in which users update their opinions when exposed to viral content?

Our Contribution:

The Spread–Acknowledge Model

- We introduce the **Spread–Acknowledge Model**
- The model **allows to quantify** how much **viral contents** impacts user opinions
- Combines the independent cascade and FJ models

- Nodes spread content similar to the independent cascade model
- When nodes are **exposed** to the content, they might **change** their innate opinion
- The content and the opinion updates trickle through the network

- The model proceeds in **rounds**
- In each round, the nodes first **update their opinions** and then **share content**

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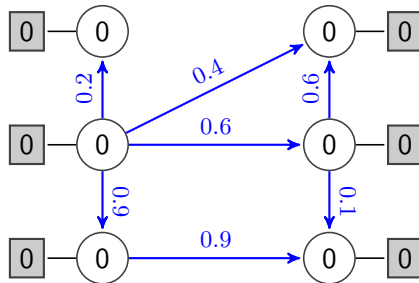
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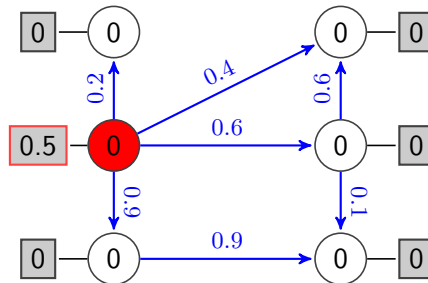
- Initially, all nodes have innate and expressed opinion 0






○ The node is not influenced.

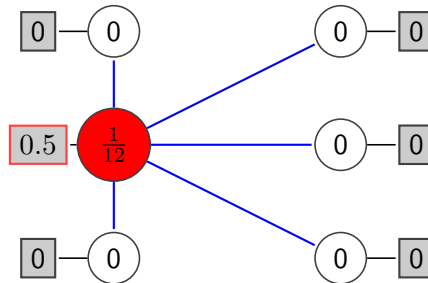
- After being **exposed** to the viral content, the red node **changes** its innate opinion and spreads information to its neighbors

-  The node changes its innate opinion, and spreads the content.
-  The node changes its innate opinion at this step.
-  The node is not influenced.



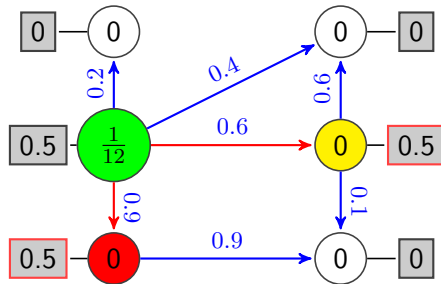
- The **expressed** opinions get updated as a consequence of the **changed innate** opinions

-  The node changes its innate opinion, and spreads the content.
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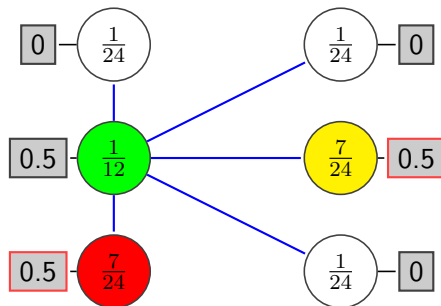
- The **green** node shares the content
- The **yellow** node changes its innate opinion (but does *not* share it)
- The **red** node changes its innate opinion and shares the content in the next round

- The node only changes its innate opinion.
- The node already spreads information.
- The node changes its innate opinion, and spreads the content.
- The influencing edge.



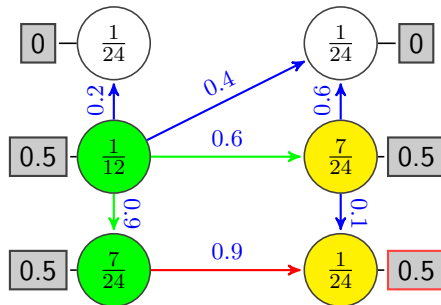
- As the innate opinions get updated, the expressed opinions are also updated

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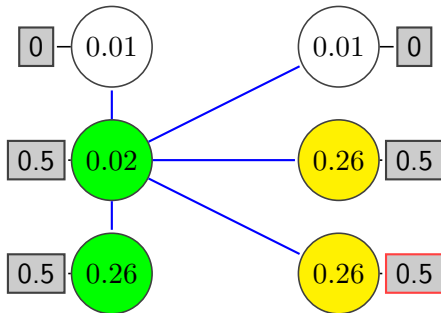
- The bottom-left node **influences** the bottom-right node
 The bottom-right node **changes** its innate opinion, but it does not spread the content

- The node only changes its innate opinion.
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- When no more contents get shared, the expressed opinions are updated as in the FJ model

- The node only changes its innate opinion.
- The node already spreads information.
- The node changes its innate opinion, and spreads the content.
- The influencing edge.



When nodes acknowledge the viral content they update their innate opinions:

- Let $\epsilon > 0$ be a parameter
- Marketing content
 - Acknowledging users **increase** their opinion
 - New opinion $s'_v = s_v + \epsilon$
- Polarizing content
 - Proponents **like it more**, opponents are **repelled**
 - Let $\tau > 0$ be a parameter
 - New opinion $s'_v = \begin{cases} s_v + \epsilon, & \text{if } s_v \geq \tau \\ s_v - \epsilon, & \text{if } s_v < \tau \end{cases}$

⇒ In the experiments, we will study how these different types of contents influence polarization, disagreement, ...

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Simulating the Model

How can we efficiently simulate our model?

■ Previously:

- Friedkin–Johnsen model has nice closed form solution $z = (I + L)^{-1}s$
- Independent cascade model can be simulated using reverse reachable sets

■ Now:

- Opinion formation and information spreading are **interleaved**
 - Not clear how to use existing techniques for our model
- How can we efficiently simulate our model?

Update opinions

$$z_i^1 = \frac{s_i^0 + \sum_{j \in N(i)} w_{i,j} z_j^0}{1 + \sum_{j \in N(i)} w_{i,j}}$$



Round 1:

Information spread; Nudge innate opinions

$$s_i^1 = f_0(s^0)$$

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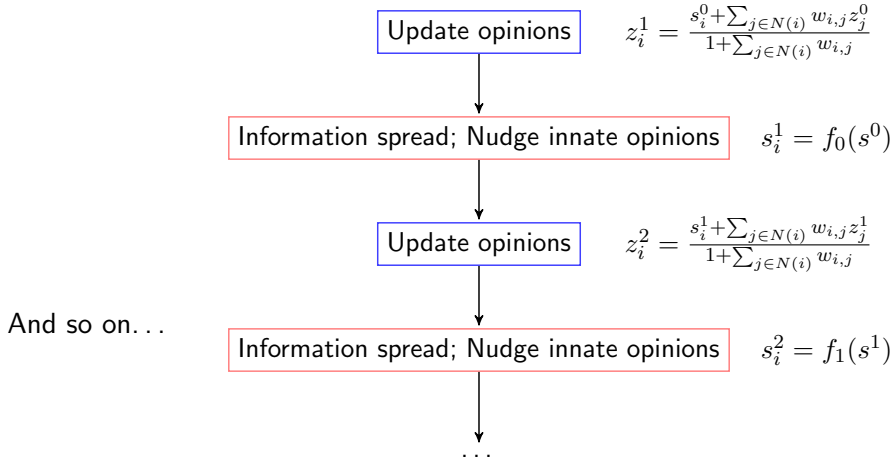
Update opinions

$$z_i^2 = \frac{s_i^1 + \sum_{j \in N(i)} w_{i,j} z_j^1}{1 + \sum_{j \in N(i)} w_{i,j}}$$

Round 2:

Information spread; Nudge innate opinions

$$s_i^2 = f_1(s^1)$$



- In our model the information spreading and the opinion updates are **interleaved**
 - Consider the simpler **Two-Stage Model**:
 - **Stage 1**: Run the independent cascade model
 - this reveals which users change their innate opinions
 - **Stage 2**: Run the FJ model with the updated innate opinions
- ⇒ can be simulated using known results
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- **Lemma**: The Spread–Acknowledge Model and the Two-Stage Model **generate the same distributions** over the updated innate and expressed opinions.
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Further Results

- We give algorithms for the **efficient simulation** of our model
- We **prove** their correctness and error guarantees

- We give approximation algorithms for **maximizing the sum of opinions**
- Builds upon framework of reverse reachable sets (Borgs et al.; 2014)
- We give heuristics for **maximizing disagreement, polarization, ...**

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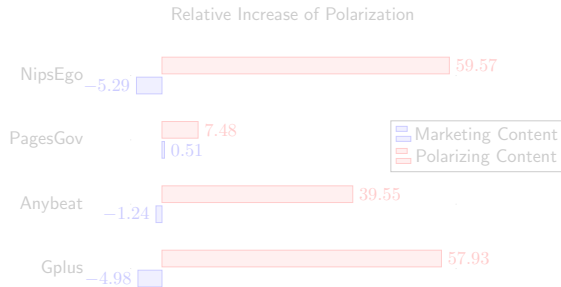
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Experiments

- Polarization is the variance of the opinions

→ $\mathcal{P} = \sum_{i \in V} (z_i - \bar{z})^2$ where $\bar{z} = \frac{1}{|V|} \sum_i z_i$ is the average opinion

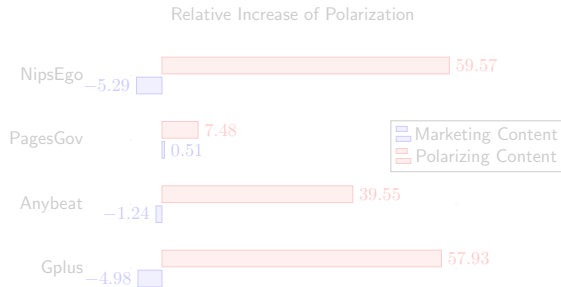
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- Marketing contents and polarizing contents have very different impacts!



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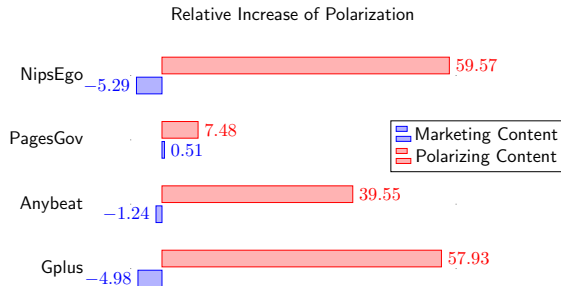
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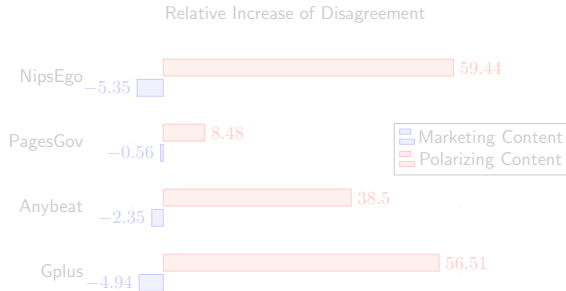
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- Disagreement measures how much interacting users disagree

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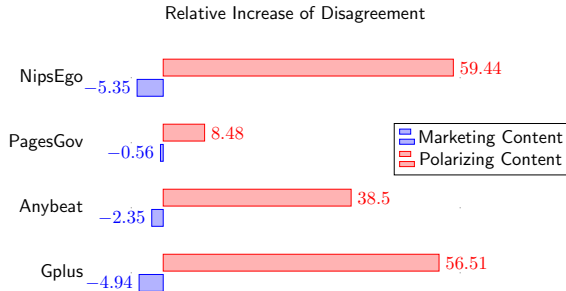
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Summary and Future Work

Our contributions:

- Novel model that allows to quantify how viral content influences user opinions
- Algorithms to efficiently simulate it
- Experiments show big difference between controversial marketing contents

Interesting directions for future work:

- Validating the model in practice
- Estimating the model parameters
- Can we get approximation algorithms for optimizing polarization, disagreement?

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